

AP Calc BC
Chapter 2 Review Problems

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2.1

① @ $P = (15, 250)$

$$\frac{694 - 250}{5 - 15} = -44.4 \text{ gpm}$$

$$\frac{111 - 250}{20 - 15} = -27.8 \text{ gpm}$$

$$\frac{444 - 250}{10 - 15} = -38.8 \text{ gpm}$$

$$\frac{28 - 250}{25 - 15} = -22.2 \text{ gpm}$$

$$\frac{0 - 250}{30 - 15} = -16.6 \text{ gpm}$$

⑥ $P'(15) \approx \frac{-38.8 + -27.8}{2} = \boxed{-33.3 \text{ gpm}}$

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2.1

⑤ $y = x^2 + 2x$ at the point $(-3, 3)$

⑥ $\lim_{x \rightarrow -3} \frac{(x^2 + 2x) - (-3)}{x - (-3)} = \lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{x + 3} =$

$$\lim_{x \rightarrow -3} \frac{(x+3)(x-1)}{(x+3)} = \lim_{x \rightarrow -3} x-1 = \boxed{-4}$$

$$\boxed{y'(-3) = -4}$$

⑥ $\lim_{h \rightarrow 0} \frac{((x+h)^2 + 2(x+h)) - (x^2 + 2x)}{h} =$

$$\lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h} =$$

$$\lim_{h \rightarrow 0} \frac{2xh + h^2 + 2h}{h} = \lim_{h \rightarrow 0} \frac{h(2x + h + 2)}{h} =$$

$$\lim_{h \rightarrow 0} 2x + h + 2 = 2x + 2 \quad y'(x) = 2x + 2$$

$$y'(-3) = -6 + 2$$

$$\boxed{y'(-3) = -4}$$

⑥ $\boxed{y - 3 = -4(x + 3)}$

⑦

⑦ $y = \frac{2}{x+3}$ at $x=a$

$$\lim_{x \rightarrow a} \frac{\frac{2}{x+3} - \frac{2}{a+3}}{x-a} = \lim_{x \rightarrow a} \frac{2a+6 - 2x-6}{(x+3)(a+3)} \frac{1}{x-a}$$

$$= \lim_{x \rightarrow a} \frac{2a - 2x}{(x+3)(a+3)} \cdot \frac{1}{x-a} = \lim_{x \rightarrow a} \frac{2(a-x)}{(x+3)(a+3)(x-a)}$$

$$= \lim_{x \rightarrow a} \frac{-2}{(x+3)(a+3)} = \boxed{\frac{-2}{(a+3)^2}}$$

⑧ ⑤ $v_0 = 0$ ⑥ C, since slope of tangent there is steeper.

⑦ A = speeding up (inc & cu)

B = slowing down (inc & CD)

C = neither (inc, but changing concavity)

⑨ The position of the car did not change.

25) C = cost in dollars
 x = units

$$C(x) = 5000 + 10x + 0.05x^2$$

$$\textcircled{a} \quad \frac{C(105) - C(100)}{105 - 100} = \frac{\$20.25}{\text{unit}}$$

$$\textcircled{b} \quad \lim_{x \rightarrow 100} \frac{C(x) - C(100)}{x - 100}$$

$$\frac{C(101) - C(100)}{101 - 100} = \frac{\$20.05}{\text{unit}} \lim_{x \rightarrow 100} \frac{0.05x^2 + 10x - 1500}{x - 100}$$

$$\lim_{x \rightarrow 100} \frac{0.05(x^2 + 200x - 30,000)}{(x - 100)}$$

$$\lim_{x \rightarrow 100} \frac{0.05(x-100)(x+300)}{(x-100)} =$$

$$\lim_{x \rightarrow 100} 0.05(x+300) = \frac{1}{20}(400) = \textcircled{20 \frac{\$}{\text{unit}}}$$

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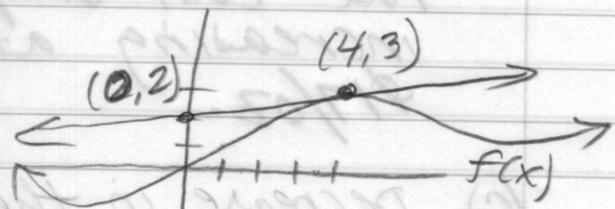
2.8

$$\textcircled{3} \quad g'(0), 0, g'(4), g'(2), g'(-2)$$

$$g'(0) < 0, \quad g'(-2) \approx 2, \quad g'(2) \approx 1, \quad g'(4) \approx \frac{1}{2}$$

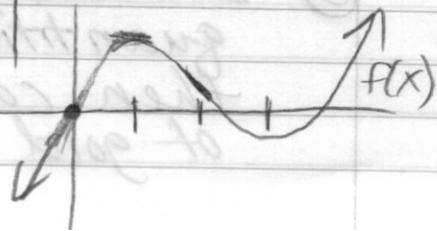
$$\textcircled{4} \quad f(4) = 3$$

$$f'(4) = \frac{3-2}{4-0} = \frac{1}{4}$$



$$\textcircled{5} \quad f(0) = 0 \quad f'(1) = 0$$

$$f'(0) = 3 \quad f'(2) = -1$$



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$$\textcircled{19} \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h}$$

$$f(x) = \sqrt{x}$$

$$x = 1$$

$$\textcircled{20} \lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h}$$

$$f(x) = x^3$$

$$x = 2$$

$$\textcircled{21} \lim_{x \rightarrow 1} \frac{x^9 - 1}{x - 1}$$

$$f(x) = x^9$$

$$x = 1$$

$$\textcircled{22} \lim_{x \rightarrow 3\pi} \frac{\cos x + 1}{x}$$

$$f(x) = \cos x$$

$$x = 3\pi$$

$$\textcircled{23} \lim_{t \rightarrow 0} \frac{\sin(\frac{\pi}{2} + t) - 1}{t}$$

$$f(x) = \sin x$$

$$x = \pi/2$$

$$\textcircled{24} \lim_{x \rightarrow 0} \frac{3^x - 1}{x}$$

$$f(x) = 3^x$$

$$x = 0$$

$\textcircled{27} C(x)$ = cost in dollars of producing gold
 x = ounces of gold

a) $C'(x)$ = rate of change in production cost per ounce of gold in $\$/\text{oz}$.

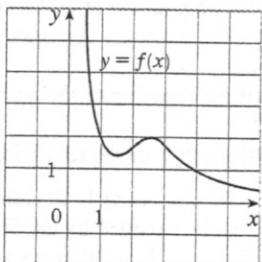
b) $C'(800) = 17$ means that when 800 oz of gold are produced the cost of production is increasing at a rate of $\$17/\text{oz}$.

c) decrease in the short term w/ high quantities of gold to produce, then cost will increase as quantities of gold become more scarce.

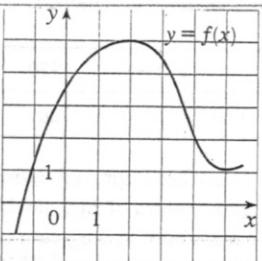
2.9**Exercises**

- 1–3 □ Use the given graph to estimate the value of each derivative. Then sketch the graph of f' .

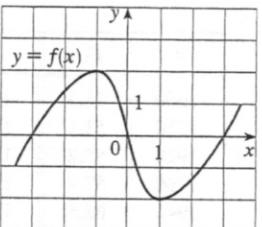
1. (a) $f'(1)$
 (b) $f'(2)$
 (c) $f'(3)$
 (d) $f'(4)$



2. (a) $f'(0)$
 (b) $f'(1)$
 (c) $f'(2)$
 (d) $f'(3)$
 (e) $f'(4)$
 (f) $f'(5)$

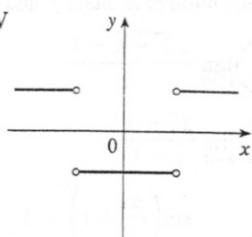
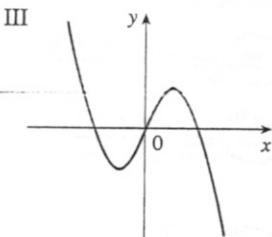
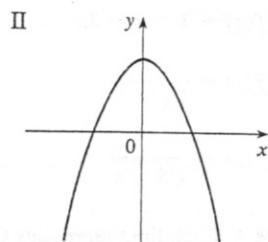
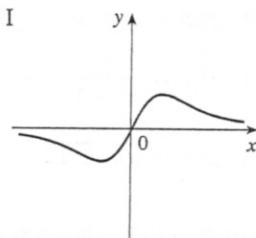
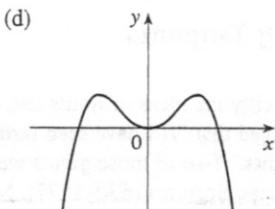
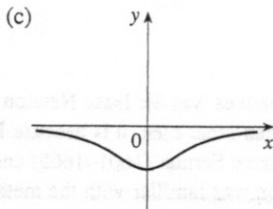
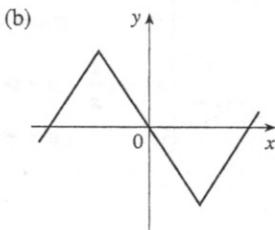
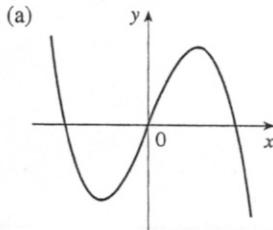


3. (a) $f'(-3)$
 (b) $f'(-2)$
 (c) $f'(-1)$
 (d) $f'(0)$
 (e) $f'(1)$
 (f) $f'(2)$
 (g) $f'(3)$

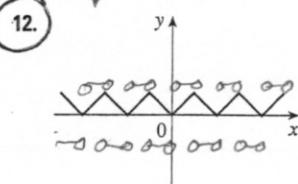
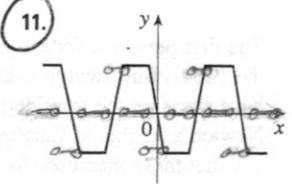
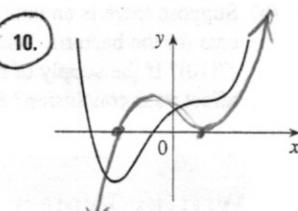
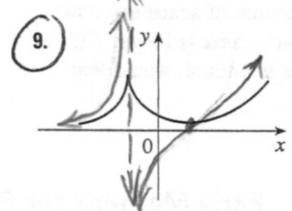
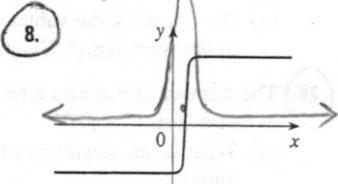
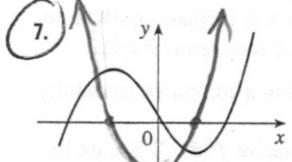
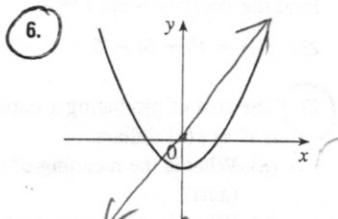
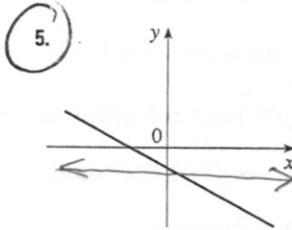


4. Match the graph of each function in (a)–(d) with the graph of its derivative in I–IV. Give reasons for your choices.

Resources / Module 3 / Derivatives as Functions / Problems and Tests



- 5–13 □ Trace or copy the graph of the given function f . (Assume that the axes have equal scales.) Then use the method of Example 1 to sketch the graph of f' below it.



(28) $n(t)$ = number of bacteria
 t = hours

(a) $n'(5)$ = the rate of change of bacteria per hour after 5 hours.

(b) $n''(10) > n'(5)$ if space & resources are unlimited

Rate of growth would slow down eventually if space & resources are unlimited.

(30) $C(t)$ = cash per capita in circulation
 t = time (years)

$$C'(1980) \approx \frac{1063 - 265}{1990 - 1970} = \frac{\$39.9 \text{ per capita}}{\text{year}}$$

This means that in 1980, cash per capita is increasing at a rate of \$39.90 per person per year.

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(31) (a) $y = 9 - 2x^2$ at $(2, 1)$

$$\lim_{x \rightarrow 2} \frac{(9 - 2x^2) - (9 - 8)}{x - 2} = \lim_{x \rightarrow 2} \frac{9 - 2x^2 - 1}{x - 2} =$$

$$\lim_{x \rightarrow 2} \frac{8 - 2x^2}{x - 2} = \lim_{x \rightarrow 2} \frac{2(4 - x^2)}{x - 2} = \lim_{x \rightarrow 2} \frac{2(2 - x)(2 + x)}{x - 2}$$

$$\lim_{x \rightarrow 2} -2(2 + x) = -8 \quad [y - 1 = -8(x - 2)]$$

$$(x)^2 = 8 - 2x^2$$

$$39) (a) s(t) = 1 + 2t + \frac{t^2}{4}$$

$$\frac{s(3) - s(1)}{3-1} = \frac{s(2) - s(1)}{2-1} =$$

$$\frac{s(1.5) - s(1)}{1.5-1} = \frac{s(1.1) - s(1)}{1.1-1} =$$

$$6) \lim_{t \rightarrow 1} \frac{(1+2t+\frac{t^2}{4}) - (1+2+\frac{1}{4})}{t-1} =$$

$$\lim_{t \rightarrow 1} \frac{\frac{1}{4}t^2 + 2t - \frac{9}{4}}{t-1} = \lim_{t \rightarrow 1} \frac{t^2 + 8t - 9}{4(t-1)} =$$

$$\lim_{t \rightarrow 1} \frac{(t+9)(t-1)}{4(t-1)} = \frac{10}{4} = \boxed{5/2} \text{ m/s}$$

$$41) f'(3) > f'(2) > f'(5) > 0$$

$f''(5) < 0$ since f is concave down
there

$$f''(5), 0, f'(5), f'(2), 1, f'(3)$$

$$f'(5) \approx 1/4 \quad f'(2) \approx 2/3 \quad f'(3) \approx 2$$

$$42) f(x) = x^3 - 2x \quad 2) f'(2) = 10 \quad b) \boxed{y-4=10(x-2)}$$

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 - 2(x+h) - x^3 + 2x}{h} =$$

$$\lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 2x - 2h - x^3 + 2x}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{x}(3x^2 + 3xh + h^2 - 2)}{\cancel{x}} = \boxed{3x^2 - 2 = f'(x)}$$

$$44) \lim_{h \rightarrow 0} \frac{(2+h)^6 - 64}{h} = f'(a)$$

$$a=2 \quad f(x) = x^6$$

45) $C(r) =$ cost of repaying a student loan
 $r = \%$ interest per year

a) $C'(r)$ is the rate of change of the cost of repaying a student loan in $\$/\%$ interest.

b) $C'(10) = 1200$ means that when the interest rate is 10%, then the cost of repaying the loan is increasing at a rate of \$1200 per percent interest per year.

c) $C'(r)$ is always positive because C will increase as r increases.

$$49) a) f(x) = \sqrt{3-5x}$$

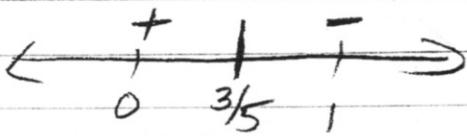
$$\lim_{h \rightarrow 0} \frac{(\sqrt{3-5(x+h)} - \sqrt{3-5x})(\sqrt{3-5x-5h} + \sqrt{3-5x})}{h} =$$

$$\lim_{h \rightarrow 0} \frac{(3-5x-5h) - (3-5x)}{h(\sqrt{3-5x-5h} + \sqrt{3-5x})} = \lim_{h \rightarrow 0} \frac{-5h}{h(\sqrt{3-5x-5h} + \sqrt{3-5x})}$$

$$\lim_{h \rightarrow 0} \frac{-5}{\sqrt{3-5x-5h} + \sqrt{3-5x}} = \boxed{\frac{-5}{2\sqrt{3-5x}} = f'(x)}$$

$$\textcircled{6} \quad f(x) = \sqrt{3-5x} \quad f'(x) = \frac{-5}{2\sqrt{3-5x}}$$

$$3-5x \geq 0$$



$$x \in (-\infty, \frac{3}{5}]$$

$$x \in (-\infty, \frac{3}{5}]$$

\textcircled{c} on calculator

\textcircled{51} $f(x)$ is not differentiable at

* $x=-4$ because it isn't continuous there

* $x=-1$ because there is a cusp in the graph

* $x=2$ because f isn't continuous there

* $x=5$ because f has a vertical tangent line.

$$\textcircled{52} \quad \textcircled{a} \quad F'(1950) \approx \frac{1 \text{ child}}{10 \text{ years}} \quad F'(1987) \approx \frac{1 \text{ child}}{20 \text{ years}}$$

$$F'(1965) \approx -\frac{1 \text{ child}}{10 \text{ years}}$$

\textcircled{b} These represent how the total fertility rate is increasing or decreasing at certain times in history.

\textcircled{c} Factors can include economic growth or slowing, foreign relations and war & cultural shifts.